

Absolute Return Stock Basket



Imagine an index fund that mirrors the index during bullish market phases, but pulls out of the market and waits during bearish phases until prices go up again. The idea is simple, but most fund companies fail in implementing it. Imagine an indicator that tells you when to invest how much in which stocks in order to beat the underlying index; a nice thought or a realistically achievable goal?

This article will take you away from the market for a little while to the hallways of Bell Labs during the year 1956, in order to introduce a new concept, allowing you to beat any stock index by re-weighting the stocks of the index in a portfolio. Risk shall be kept at a minimum but performance will not suffer because of it.



Lessons from the Realm of Information Technology

Let us assume that just before the market closed everyday you could contact an oracle that would reveal to you how a certain stock would perform the following day. Whether it goes up or down; explodes or crashes. With an information source such as that, it would be easy to earn money in the markets. You only need to put all of your capital today in the winning stock of tomorrow, thereby achieving the best possible performance. But what if the oracle was right most of the time, but about 10% of the time would deliver a false forecast. Would it still be a good idea to invest all of your capital in the security recommended by the oracle? After all, you would be right 90% of the time. What can the 10% rate of failure do to you? But what happens when you lose all of your capital because of one wrong forecast and then are unable to profit from the following correct forecasts?

That is the question presented to the scientists at Bell Laboratories. Their jobs however do not involve the stock market, but instead with improving signal quality in noisy telephone lines. But the situation was similar in a way. In a static-filled telephone line, it can be assumed that the receiver will understand the same thing the sender has spoken and that a bit sent by the sender will also be the same bit arriving at the receiver. But it is not 100% reliable. So the question was; how strongly should the receiver trust the information he receives as being the same sent by the sender? That riddle led to the birth of a new science, which to this day influences all areas of information processing. It is called Information Technology.

Money Management – Just a Game

I do not want to burden you too much with the story behind the science. This is an article about the markets. Therefore, I invite you to take part in a little game of dice. It will help clarify the question of the all deciding “how much” with the help of a practical example. You receive €100

starting capital. Now the dice is thrown. If it shows a number greater than 2 then you win and you double your money. But if the dice shows 1 or 2 then I win and get to keep the amount you have bet.

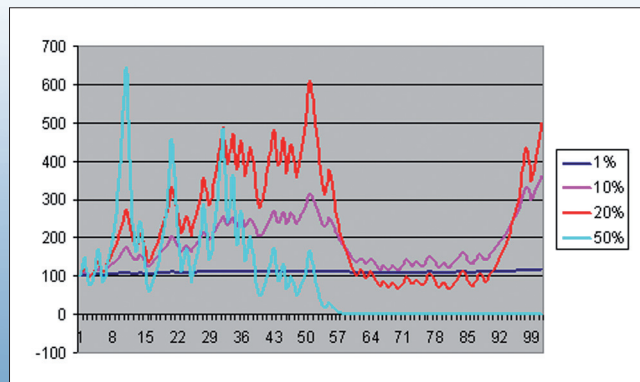
It is a game that you actually should not lose. The odds are 2 to 1 in your favour with each throw of the dice. You can only lose this game if you rely too strongly on your advantage. If you bet 100% of your capital then it could be that you lose all of your capital immediately. If you bet 50% of your capital then a small series of losses will relieve you of all your capital as well (by the way, betting in full Euro amounts only). If however you bet 1% of your capital then you will probably win far fewer Euros as would be the case with an optimum bet size.

Figure 1 shows the various results of the game depending on how much you risk per throw of the dice. If you risk only 1% of your available capital with every throw, you hardly win anything, but you also do not lose much. However if you risk 50% of your money with every play the strategy looks good in the beginning, but you will lose all of your capital in any short stretch of bad luck, even though you should win with odds of 2 to 1 (an excel file of the game can be downloaded at <http://quanttrader.at/traders/spiel.xls>).

John Kelly Jr.'s Brilliant Solution

That brings us back to Information Technology. In the year 1956, scientist John Kelly Jr. posed the question of whether it would be possible to calculate the ideal amount for betting in such a game of dice. How much should one trust an insecure piece of information when you know that chances are generally in your favour, but there is a chance of experiencing several unfavourable events? Initially, Kelly did not involve the markets in his studies. This was considered below the level of a mathematician at Bell Labs. However, others soon recognised the question's implications.

F1) Game Results with Various Betting Amounts



A game of dice you can only win - except if you bet too much. You double your money if the dice turns up a number greater than 2. You lose the bet if you role the numbers 1 or 2. What percentage of your capital should you bet in each throw in order to attain maximum profits after 100 throws? The question can be answered with the help of the Kelly formula.

I do not want to bore you with the mathematics behind Kelly's discovery. If interested you can read his original research paper "A New Interpretation of Information Rate" at <http://www.bjmath.com/bjmath/kelly/kelly.pdf>. Instead I will show you the results of the study and their application in the markets. Kelly found a simple relation that allows calculating the optimal capital amount to risk for maximum profit growth. The formula is simply called "edge/odds": i.e. that a person should risk the exact amount of capital, which is equal to his advantage.

For investment purposes the formula is presented somewhat differently, but it has the same meaning. The percentage of playing capital which one should risk for maximum possible profit growth is calculated as follows:

$$\%Win - [\%Lose / (avg.Win/avg.Lose)]$$

%Win is the probability of a profit, %Lose is the probability of a loss, avg.Win is the average profit and avg.Lose is the average loss.

Now what does the formula mean in practice? Let us examine what happens when a variable within the formula changes. In that way you can quickly recognise what the formula is saying and understand what leads us further on the way to an Absolute Return stock portfolio.

The Kelly formula calculates how much capital should be risked with every new play of the game. This percentage increases as the probability of a profit increases, because %Win is in the fraction's numerator. On the other hand the percentage of capital to be risked is reduced as the probability of profit decreases and average loss in relation to average profit increases.

This seems evident: If you have a low trade hit rate or a high average loss, then less should be invested. If you have small losses and a high probability of profit, then more should be risked in order to optimise success.

How would this formula look with our little game of dice? In this case %Win equals 66% since 4 out of 6 possible events are winners (4/6=66%). Accordingly %Lose equals 33% (2/6= 33%) or (100%-33%). The average profit is exactly as high as the average loss because you either win the amount you bet or you lose it. Accordingly avgWin/avg.Lose is equal to 1.

If you insert the numbers into the formula it results in the following: $0.66 \cdot 0.33 \cdot 1 = 0.33$. This means that with every throw of the dice you risk 33% of your capital in order to get the maximum results out of the game (since a bet is limited to whole Euro amounts, one is better off in practice to keep bets a little under 33%).

Portfolio Application

The Kelly formula serves as the basis for numerous fractional money management methods. The formula is wide spread among the Black Jack community; a game with a small possible statistical advantage for the player and an identifiable loss-risk and profit opportunities. But this is not about the game of Black Jack, but instead how to outperform a buy and hold index basket strategy with the help of the formula.

Thereby it is helpful to understand a buy and hold strategy as a sequence of virtual day trades. For example, if you hold a stock for one week and it increases in price from 100 to 105, then it could be a result of the following price action: 100 – 99 – 102 – 103 – 104 – 105. Or it could be the following price action: 100 – 115 – 107 – 106 – 105.

What differentiates the two cases? If you look at the daily price changes then you would have won nearly every day with the first price-move sequence, i.e. one losing day against 4 winning days. The maximum daily loss was one point. In the second sequence, circumstances are different. Here there are four losing days against one winning day and the maximum loss amounted to eight points.

In which of the two stocks would you prefer to put more capital? Presumably the first one; it offers high profit opportunity combined with small losses. That is not only what an experienced trader would do, but also the Kelly formula. It increases investment the higher the given profit probability and the lower the given average loss.

Of course these are only examples of price sequences. Five days of data is not nearly enough to analyse the price movement of a stock. But what happens if the period of analysis is one year? Not only that but also how many days the stock increased in price and how many days the stock decreased in price (daily close to close), as well as the average winning day and the average losing day. Now we put all of that data into the Kelly formula. Studying the chart in Figure 2 and you will see what the Kelly formula is capable of.

The weekly chart of Lufthansa spans a time period of two years. The indicator is the direct result of the Kelly formula. It calculates the positive and negative weeks over a period of 54 weeks. Risk capital in this example was set at €1,000.

Through the middle of July 2005, the stock moved with little or no trend. On July 29, 2005 there were a total of 29 positive weeks against 25 negative weeks. The average positive week created a profit of €0.226. The average negative week caused a loss of €0.251. Thus, there were 53% positive weeks and 46% negative weeks. Inserting these values into the Kelly formula results in the following:

$$\%Capital\ to\ risk = 53\% - (46\% / (0.226 / 0.251)) = 1.9\%$$

Thus 1.9% of our €1,000 in capital should be invested in this stock. This means with an average expected loss of €0.251/stock and week, a total of 76 stocks should be traded in the position (the indicator rounds up to units of 100 stocks so 100 shares were indicated).

This Kelly calculation is now conducted anew week after week resulting in the indicator movement shown in the chart. If negative weeks turn up as time progresses the statistical values become more

negative resulting in a reduction in the number of the shares being recommended for the position. A strong up trend would result in a much higher number of shares being recommended.

Ultimately, the indicator does nothing more than view price action as a game delivering an estimate for the coming week based on the performance of the past year. This works best with stocks demonstrating solid trends that produce few weeks with strong losses. In stocks that fluctuate without trend, the Kelly formula will recommend investing little to nothing. With strong linear trends it will suggest more aggressive investment. As you can see on the chart the formula reacted quickly in April 2006, right after the first week of falling prices. Subsequently, the position size is reduced drastically from 1600 to 1000 shares. This is because the number of negative weeks has increased and, above all, because this strongly negative week increases the parameter value of the average negative week. Both values are of course a part of the Kelly formula; so less is invested both with a falling trade hit rate and increasing loss days.

There is however a problem with this approach that must be mentioned. Look again at the current upward trend in Lufthansa in Figure 2 and imagine that it continues a little farther. How will the formula react when there are hardly any losing weeks in the observed time period? The answer is that it will allow position size to increase indefinitely, because it assumes you cannot lose betting on this stock. The formula always delivers the percentage of capital that should be

risked with the next trade. The number of shares to be traded is derived by dividing the amount of risk capital by the amount of the average losing day within the observed time period. If the average losing day is equal to zero (or very small) because there have been no negative days during a strong trend, then position size shoots way up.

As a trader you know that such circumstances cannot continue indefinitely. So a procedure must be developed to limit position size. The easiest way to do this is either to limit increasing position size manually or to limit the downside of the average losing-day parameter and for instance require a minimum risk of at least the average daily range. Of course that strays from the original idea, but since the formula was not intended for the markets in the first place it is up to us as speculators to find our own solutions and workarounds.

Preliminary Review

Before we see if this approach is good enough for putting together a high performance Absolute Return stock portfolio, let us summarise the material thus far. The Kelly formula calculates how much capital one should risk on a bet in a high number of individual plays of a game in order to attain a maximum return. To make the calculation you need the parameters of hit rate, average profit and average loss.

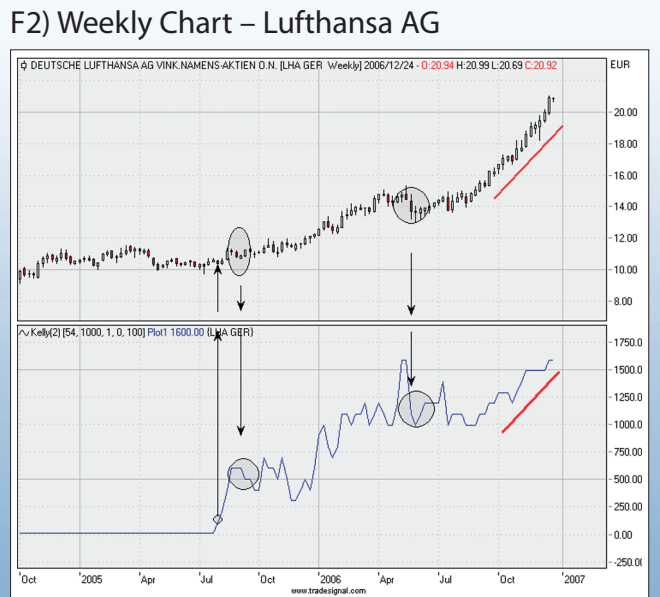
If the formula is applied to a stock, the required parameters result from the stock's daily fluctuations. These parameters are the number of positive and negative days and the average positive and negative daily returns. In essence you do nothing other than to consider the daily price moves of the stock in the same way you view the results of the throws of the dice in the game mentioned above. These values are then calculated on a daily or weekly basis. The resulting position sizes are also adjusted on a daily or weekly basis. The procedure should lead to an out performance of a buy and hold strategy.

Kelly's Approach as a Portfolio Formula

When trying the approach for position sizing on several securities you will soon notice that it works for some stocks and leads to losses in others. Nevertheless, the losses are usually smaller than the profits so that a portfolio with as many individual stocks as possible should return positive results. Without philosophising about why that is, I would like to show results for several baskets so that you can judge for yourself if the approach makes sense.

Figure 3 shows the technique applied to the stocks of the Euro Stoxx 50 index using weekly data and €1,000 risk per week and stock. The chart shows the results of the entire portfolio. The blue line represents the profit results of all completed trades; the orange line shows the total current profit/loss results of all open positions. The formula is applied to every stock contained in the index and the position recommended by the formula is taken accordingly. If the suggested position size changes then the portfolio is altered accordingly at the opening of the new week. You can see that the procedure works in practice. During strong bull phases positions are built up quickly. This can be seen by the quickly rising open profit line (orange). The blue line showing the profit results of the closed trades in contrast remains unchanged.

This is to be expected. When the formula detects a number of weeks of increasing prices with few setbacks then a lot of stocks are bought. If the rally is supported by only a few stocks, (as in mid 1998) then positions are closed and profits realised. If the market all at once

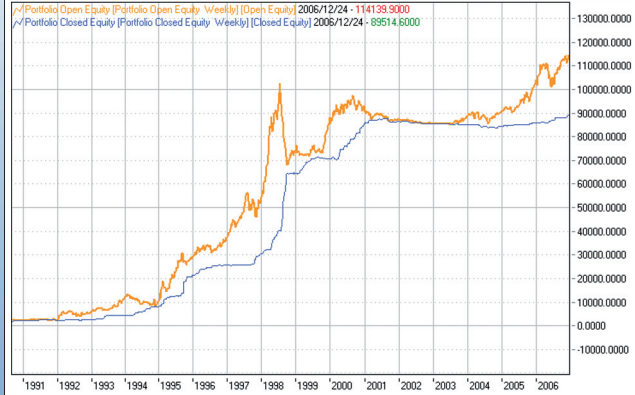


If you looked at the weekly price changes of a stock the same as you would at the performance curve of a game of luck you can deduce the ideal employment of capital. With the help of the Kelly formula you can calculate the profits and losses that can be expected. Using those numbers, the formula generates a recommended position size. This increases the more strongly the trend in the stock increases. If the trend weakens or unexpected events turn up the position is reduced. In sideways or downward trending markets the Kelly formula keeps you out of the market. The portfolio is strongly invested only if the trend is strong. The indicator shows the recommended position size if the investor is willing to risk €1,000 per week.

Source: www.tradesignal.com

F3) Kelly and the Euro Stoxx 50

| Name | Last | Period | Position | Position Size | Total Net |
|------------------------------|-------|--------|----------|---------------|-----------|
| 1 UNICREDITO ITAL. EO 0,50 | 6.62 | Weekly | Long | 400 | 9075.9000 |
| 2 ENI S.P.A. EO 1 | 25.53 | Weekly | Long | 200 | 170.1000 |
| 3 BCO SANT. CEN. HISPNEOUS | 13.98 | Weekly | Long | 160 | -219.9000 |
| 4 BCO BELVIZ ARG. HOMEO-0 | 19.28 | Weekly | Long | 160 | 62.3000 |
| 5 SUEZ S.A. EO 2 | 37.83 | Weekly | Long | 160 | -94.6000 |
| 6 SANPAOLO IMI EO 2,80 | 16.63 | Weekly | Long | 100 | -884.2000 |
| 7 IBERDROLA INH. EO 3 | 33.12 | Weekly | Long | 90 | 1630.3000 |
| 8 AMA S.A. INH. EO 2,29 | 30.37 | Weekly | Long | 80 | 723.1000 |
| 9 ENEL S.P.A. EO 1 | 7.85 | Weekly | Long | 70 | 393.0000 |
| 10 KON. PHILIPS ELECT. EO 20 | 28.11 | Weekly | Long | 60 | -114.7000 |
| 11 FORTIS | 32.33 | Weekly | Long | 60 | -19.0000 |



Respectable performance results by using the Kelly formula week for week on all of the stocks contained in the Euro Stoxx 50 index. The chart shows the performance of a Euro Stoxx 50 basket of stocks if the investor is prepared to risk €1,000 per week in every underlying stock. Investment takes place only when the individual stocks demonstrate a strong trend. During weak market phases, positions get closed. The blue line represents the profits of the closed positions, the orange line are the profits/losses of the still open positions. If the market turns down again, current accumulated profits will be realised and the investor can park his money in a money market fund.

Source: www.tradesignal.com

F4) Kelly Also Works with the MDAX



Similar performance as in the Euro Stoxx 50 was also attained with the stocks of the MDAX index of German mid-cap companies. In contrast to the Euro Stoxx 50 basket, the portfolio adjustment here took place on a daily basis. Again you can see that the approach keeps the investor away from the market during negative market phases and is strongly invested during strong trends and realises profits if pull backs occur.

Source: www.tradesignal.com

crashes then not all positions will be closed in time and a substantial part of the open profits will be given back (as in 1998). In the bear market of 2000-2003 the Kelly formula kept us away from the market. You can see in the chart that there were no open profits or losses. Therefore, there were hardly any open positions during this time. It is remarkable that a system that only trades the long side of the market survived this market phase with so few losses. After 2003 as the new bull market began new positions were put on. Since the bull market continues to this day the blue line with the profit results of the closed trades is hardly rising at this point. Should the trend change the system will close the positions that have accumulated profit since 2003 and wait for a new trend. Sounds perfect does not it?

Figure 4 shows the same procedure applied to the MDAX, this time with daily data and daily position size adjustment. It can be clearly seen that the portfolio survived the phase 2000-2003 without any major damage. Subsequently, the strong trend in 2003 was utilised well. The following sideways trend at the beginning of 2004 resulted in the positions being closed securing the accumulated profits. After the strong sell-off in 2006 the system stayed out of the market for a time. Currently (December 2006) the system is strongly invested again.

Kelly – True Portfolio Enrichment

Until now, the Kelly formula was mainly found in literature having to do with the game of Black Jack and in fractional Money Management approaches. The direct application of the formula on a stock portfolio is new and very promising. Impressive positive performance was

attained in all sampled markets. Weak market phases were survived without damage and strong upward trends were being utilised. Presumably only a few private investors will select the method for actual use. Trading a large stock portfolio is only possible with great effort and only then when transaction costs are reasonable.

It was shown that the results were better the more stocks were traded. Whether adjusting the portfolio is done on a weekly or daily basis does not seem to play a very important role. Also, the length of the observation period for the indicator is hardly a crucial factor. The results shown here with the Euro Stoxx 50 were realised with a weekly portfolio adjustment and a period observation of 54 weeks. A possible application for private investors could be to simply use the formula as an indicator of trend strength and to develop a trading approach around that.

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